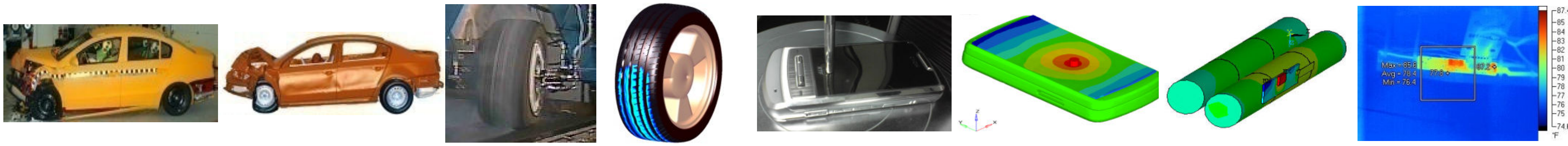

Copula-based Validation Approach for 2014 V&V Challenge Problem

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2. Uncertainties in the Challenge Problem
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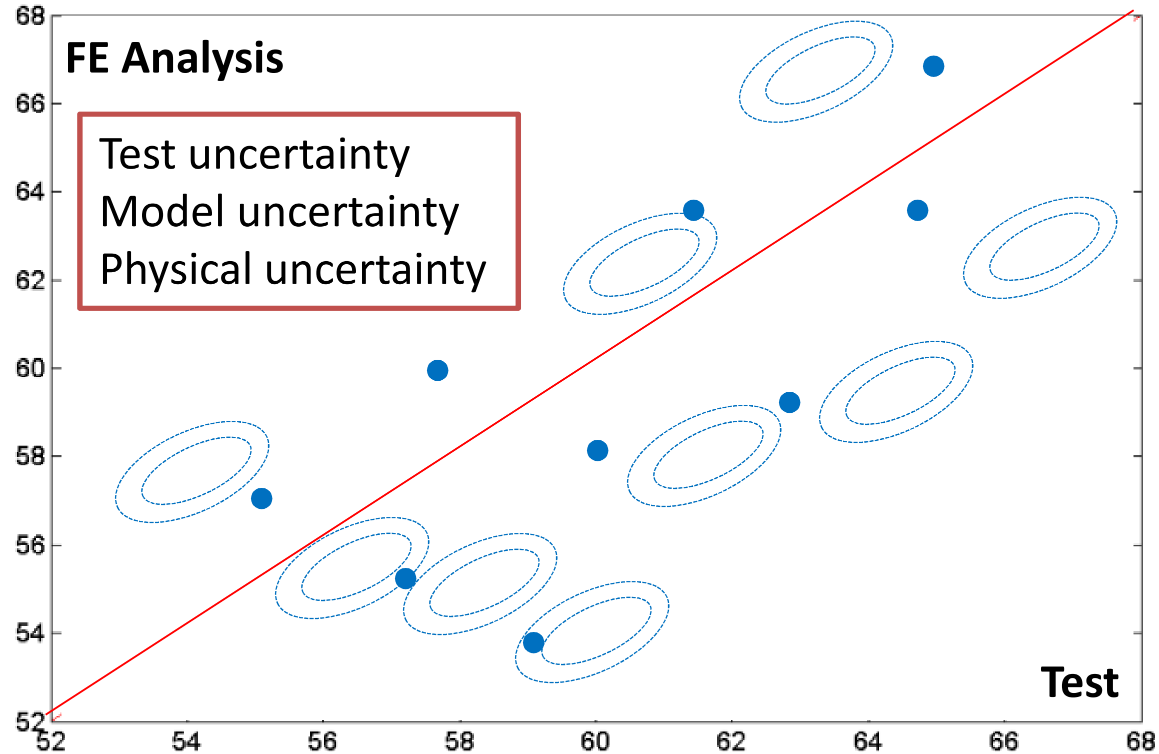
Uncertainties in Model Validation

$$(\mathbf{X})\ Y_{FE}\ \delta = Y_{Test}\ (\mathbf{X})\ \varepsilon$$

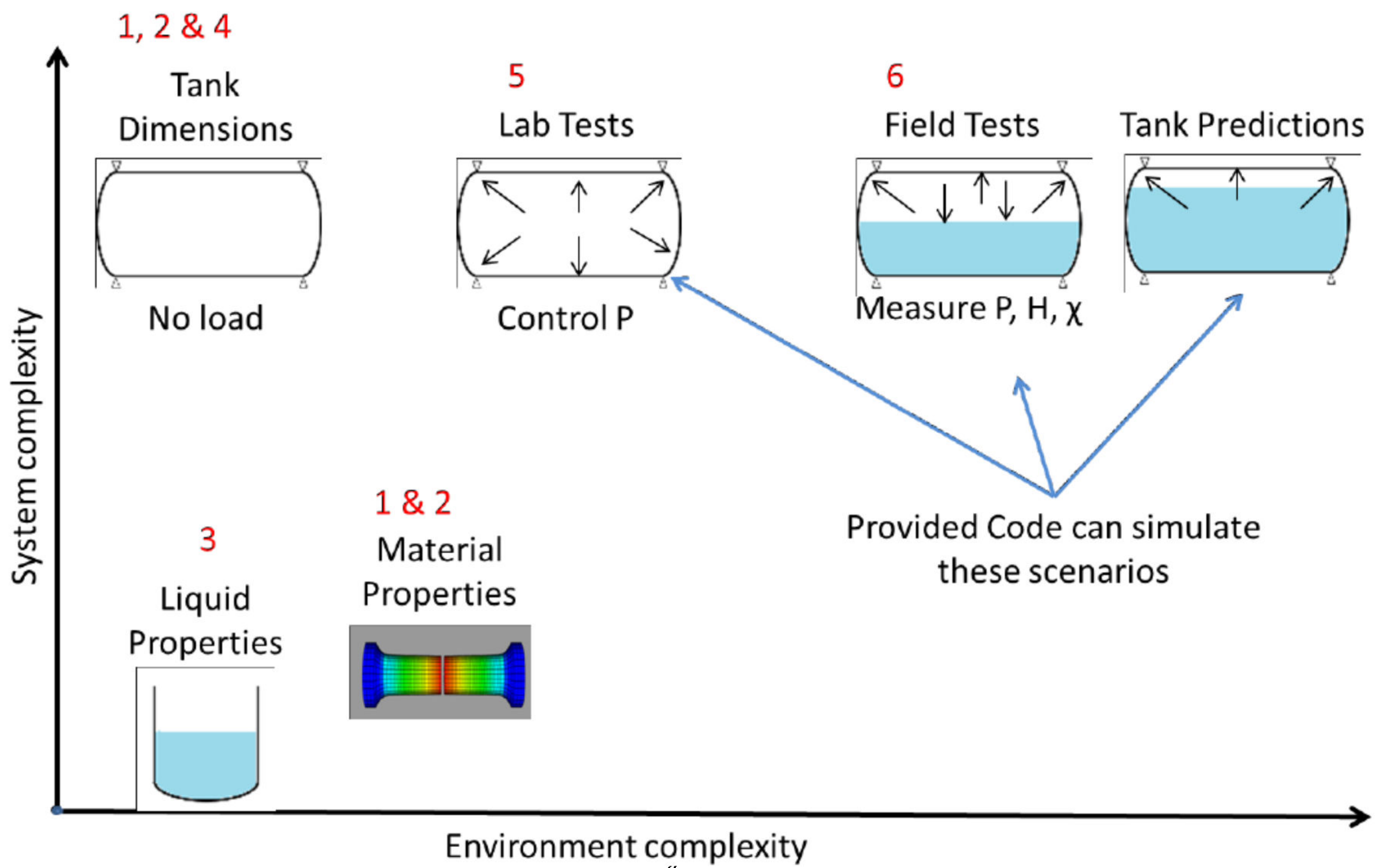
Algorithm Uncertainty



Statistical Uncertainty

Uncertainties in the Challenge Problem

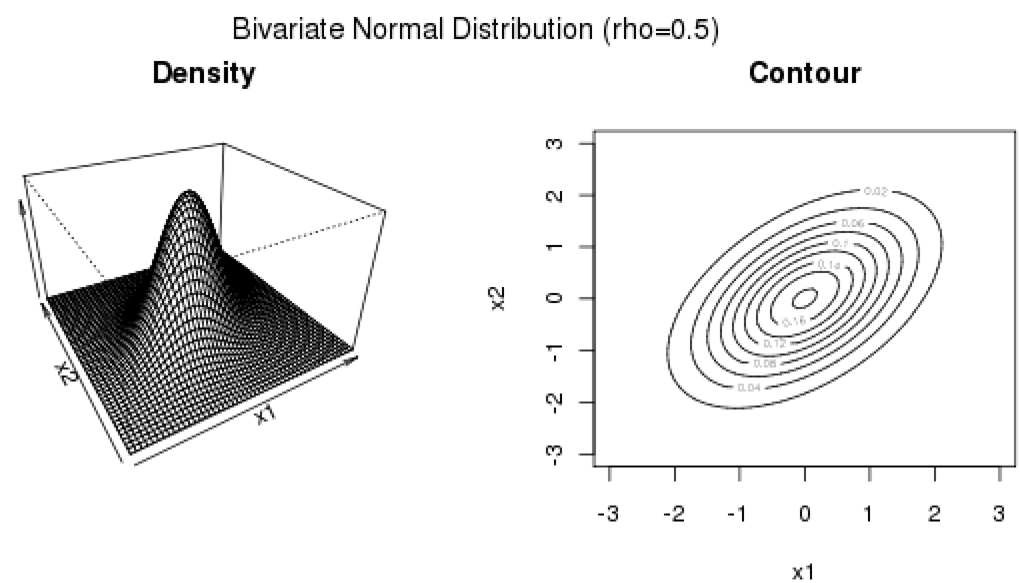
Test uncertainty; Model uncertainty; Physical uncertainty; Statistical uncertainty; Algorithm uncertainty



Copula-based Validation Approach:: what is Copula

•A general way to formulate multivariate distribution considering statistical dependence

- A copula is a joint distribution function of standard uniform random variables
- Most copulas only deal with bivariate data
- Multivariate data are often analyzed pair by pair using two-dimensional copulas



Copula-based Validation Approach::
selecting Copula

- MLE approach
  - Demands sufficient data to ensure accurate copula selection

- **Bayesian copula approach**
  - Reliable identification of true copulas even with small amount of samples

A set of hypotheses are made first as:

$$H_k\text{: } data\ come\ from\ Copula\ C_k, \qquad k = 1, \dots, Q$$

Find the copula with the highest $Pr(H_k|D)$ from a finite set of copulas

Based on Bayes' theorem, the probability that data come from the copula C_k is:

$$Pr(H_k|D) = \frac{Pr(D|H_k)Pr(H_k)}{Pr(D)} = \int_{-1}^{1} \frac{Pr(D|H_k,\tau)Pr(H_k|\tau)Pr(\tau)d\tau}{Pr(D)}$$

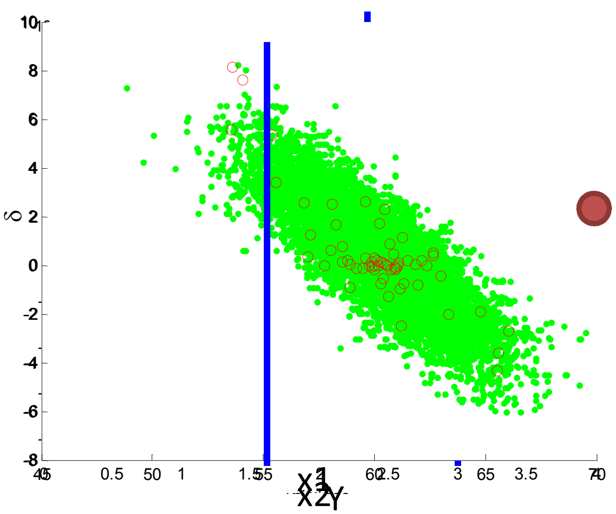
Copula-based Validation Approach (Xi et al. 2014)

A multi-dimensional joint conditional PDF is approximated by multiple two-dimensional conditional PDF

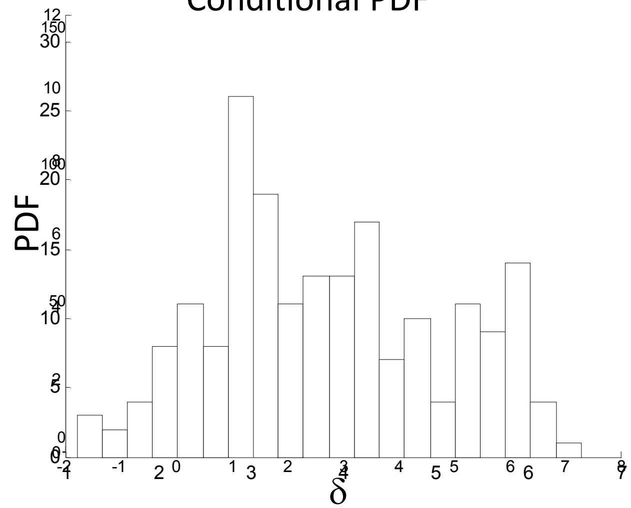
$$f\left(\delta \mid x_1 = a_1, x_2 = a_2, \ldots, x_8 = a_8, y = c\right) \cong \beta \prod_{i=1}^{8} f\left(\delta \mid x_i = a_i\right) f\left(\delta \mid y = c\right)$$

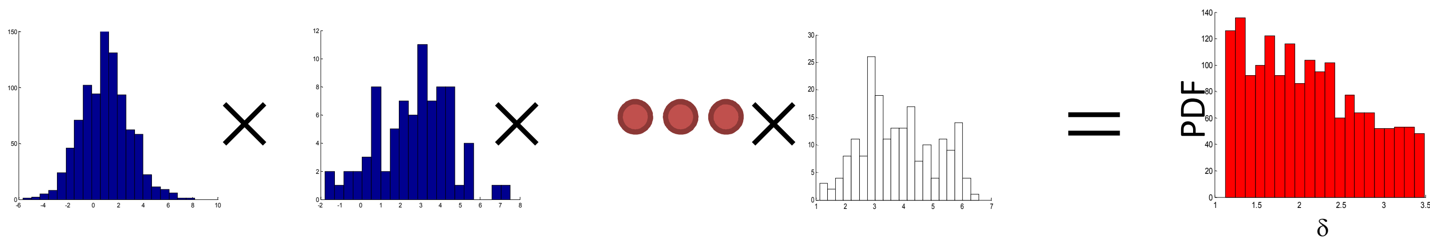
e.g. Identify model bias (mean) when
x1=2.3, x2=3.0, x3=3.0, x4=2.5, x5=4.0, x6=2.8, x7=2.6, x8=1.2, and Y = 55.

Copula model



Conditional PDF





Case Study of the Challenge Problem

Case I: compute the P.O.F based on the original simulation model

Case II: compute the P.O.F and its C.I. based on the CORRECTED simulation model

- $P = 73.5 psig$
- $\chi = 1$ ★
- $H = 50 in$

Case Study of the Challenge Problem

Case I: compute the P.O.F based on the original simulation model

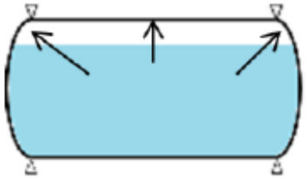
P.O.F. = Pr(maximum stress >= yield stress)

- Typical stress-strength type reliability Prediction

Max. stress = f(X)
X: physical uncertainty;
f(): simulation model (python);

Yield stress: measured quantity with uncertainty

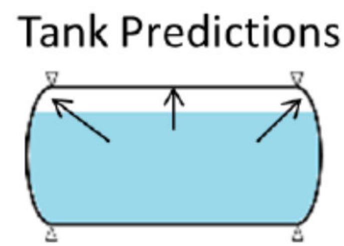
Tank Predictions



P = 73.5; H=50; χ=1

Case Study of the Challenge Problem

Case I: compute the P.O.F based on the original simulation model



P = 73.5; H=50; χ=1

X: physical uncertainty

E: Young's Modulus	10%
v: Poisson's ratio	5%
L: Tank length	1.5%
R: Tank radius	5%
T: Wall thickness	10%

uncertainty modeling of above uncertainty

Available data: i) nominal value from the manufacturer; ii) 1 or 2 sample measurements with spatial variability;

Approach 1: MLE;

Approach 2: Bayesian;

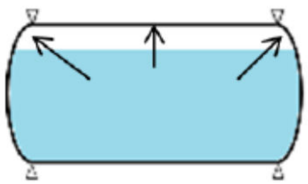
Approach 3: Interval;

Approach taken: assume normal distribution (μ = nominal value; 2σ = max. deviation from the nominal value)

Case Study of the Challenge Problem

Case I: compute the P.O.F based on the original simulation model

f(): simulation model (python)



P = 73.5; H=50; χ=1

For a given meshID, still need decide the mesh size (length, radius) to find max. stress

for a given input, change mesh size to see the changes of max. stress

5x:	1.9020e4		
10x	2.2103e4	0.5s	Decide to use 10x for the mesh
20x	2.2026e4	1.2s	size considering that yield stress
30x	2.2062e4		is about 4.5e4
40x	2.2152e4	3.6s	
50x	2.2170e4		meshID = 2
100x	2.2169e4		

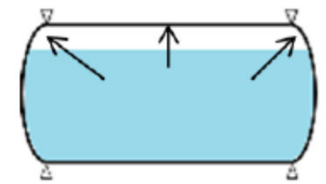
Case Study of the Challenge Problem

Case I: compute the P.O.F based on the original simulation model

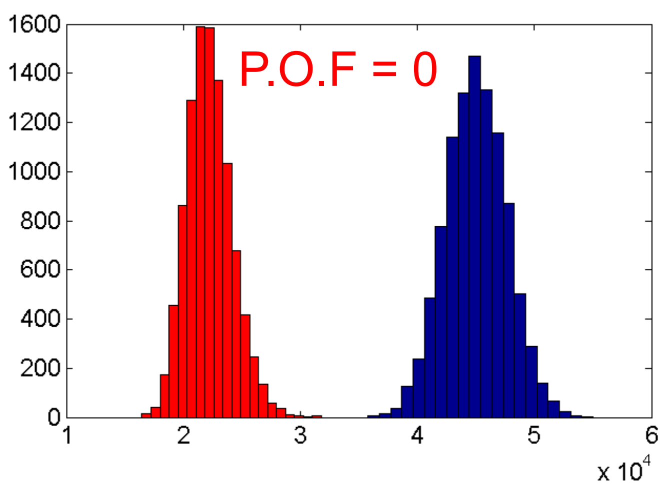
Yield stress: nominal 4.5e4 11.5% (=2σ)

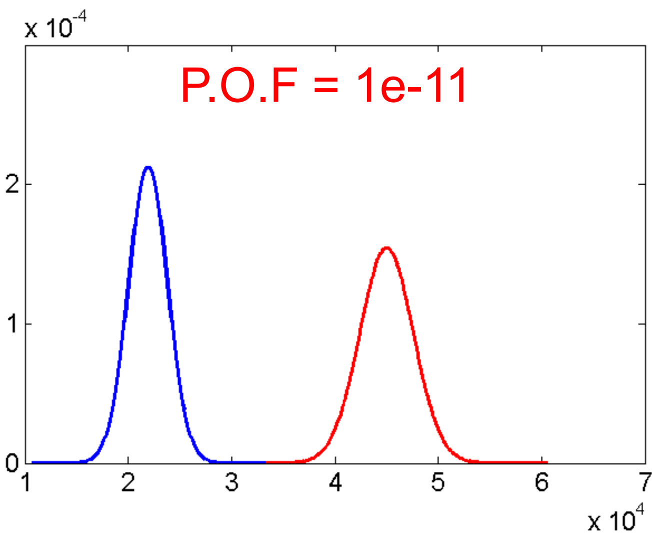
Reliability estimation algorithm:
MCS w/ 10,000 samples

Tank Predictions



P = 73.5; H=50; χ=1

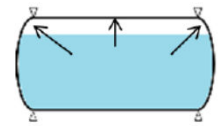




Case Study of the Challenge Problem

Case II: compute the P.O.F and its C.I. based on the CORRECTED simulation model

Tank Predictions



P = 73.5; H=50; χ=1

Step 1: Model bias of the max. disp. at required operating condition

Original model prediction of the max. disp.

Step 2: Corrected prediction of max. disp. at required operating condition

Relationship between max. disp. & stress

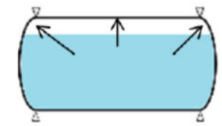
Step 3: Corrected prediction of max. stess at required operating condition

Step 4: Corrected prediction of P.O.F. with CI

Case Study of the Challenge Problem

Case II: compute the P.O.F and its C.I. based on the CORRECTED simulation model

Tank Predictions



P = 73.5; H=50; χ=1

Step 1: Model bias of the max. disp. at required operating condition

Tank 3-6: Each has different (unmeasured) material properties and dimensions

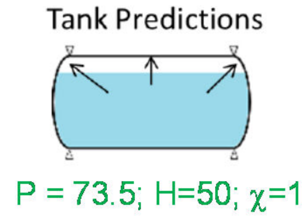
P = [30.849, 66.105, 38.194];
χ=[0.9, 0.6, 0.4];
H = [35, 40, 30];

P = 73.5; H=50; χ=1

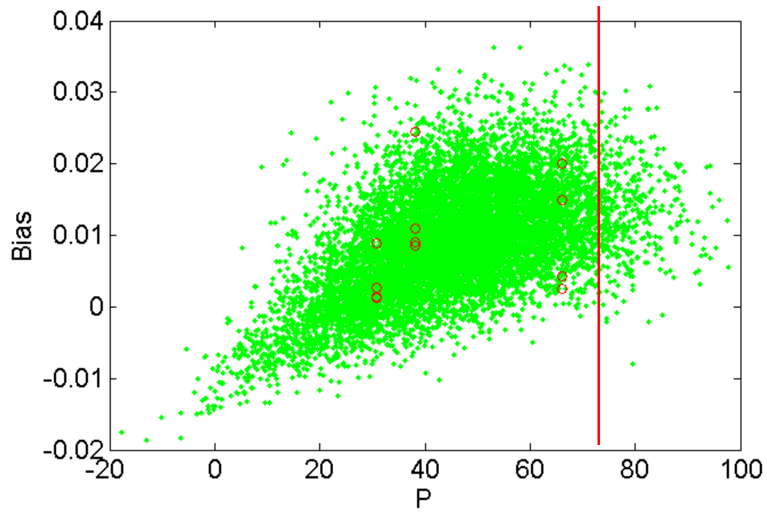
Predict bias at required operating condition given 4 repeated tests at 3 different operating conditions!

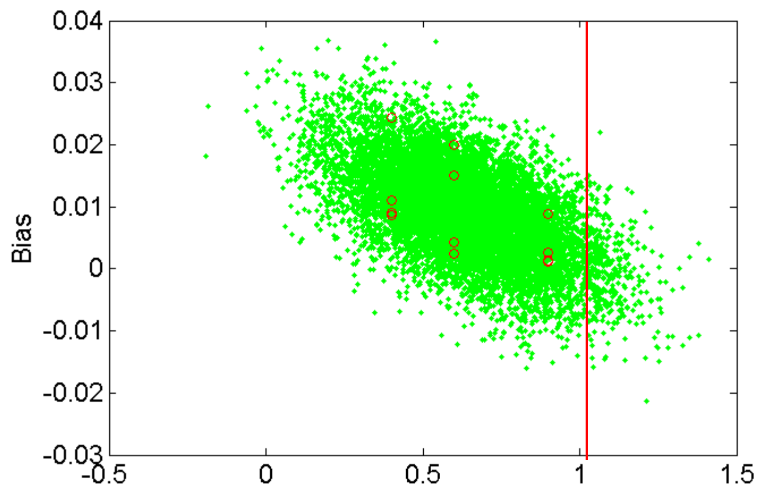
Case Study of the Challenge Problem

Case II: compute the P.O.F and its C.I. based on the CORRECTED simulation model



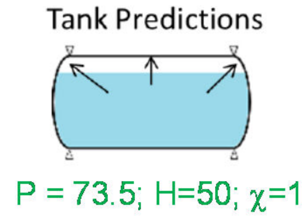
Step 1: Model bias of the max. disp. at required operating condition



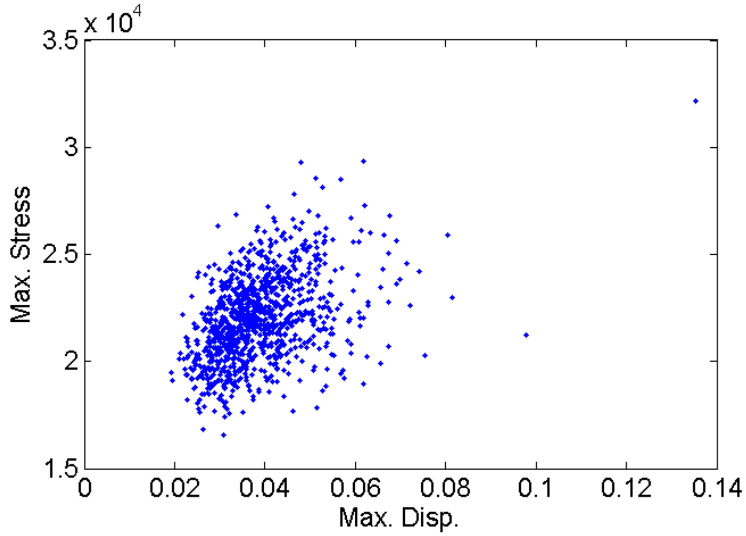


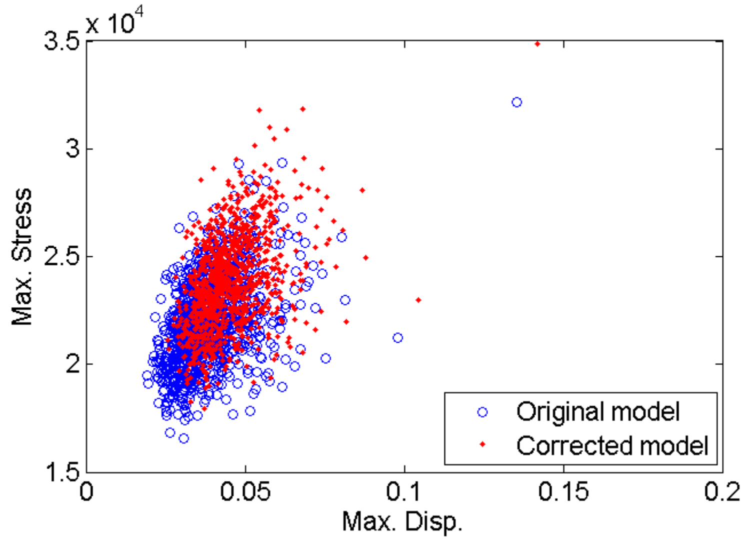
Case Study of the Challenge Problem

Case II: compute the P.O.F and its C.I. based on the CORRECTED simulation model



Step 3: Corrected prediction of max. stess at required operating condition



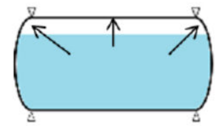


Max. stress. new = max.stress + max.stress*p*coef.

Case Study of the Challenge Problem

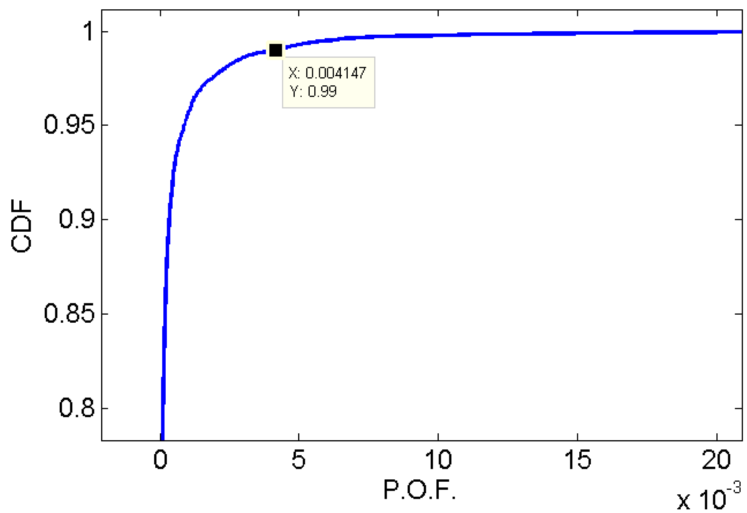
Case II: compute the P.O.F and its C.I. based on the CORRECTED simulation model

Tank Predictions

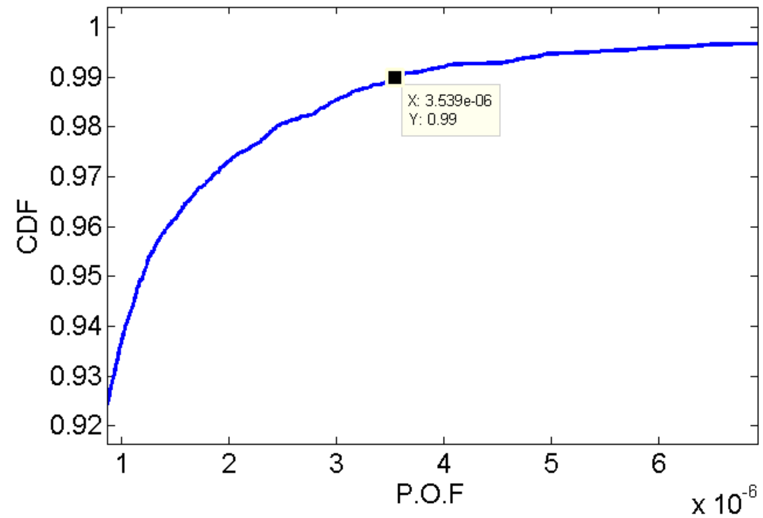


P = 73.5; H=50; χ=1

Coef. = 1



Coef. = 0.5



P.O.F <=0.004% with 99% confidence

P.O.F <=3.5e-6% with 99% confidence

Summary:: Limitations & Future Work

- Test uncertainty (measurement uncertainty)
  - Not well considered (e.g. χ)
- Model uncertainty
  - Model bias of each tank sample is uncertain due to unmeasured physical quantify
- Physical uncertainty
  - Great simplification of the uncertainty modeling
- Statistical uncertainty
  - Significant due to above simplification
- Algorithm uncertainty
  - MCS w/ 10,000 samples for reliability (or P.O.F.) calculation